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Analysis of the vertices DDV and D^*DV with light-cone QCD sum rules

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Abstract. In this article, we study the vertices DDV and D^*DV with the light-cone QCD sum rules. The strong coupling constants g_{DDV} and f_{D^*DV} play an important role in understanding the final-state re-scattering effects in the hadronic B decays. They are related to the basic parameters β and λ respectively in the heavy quark effective Lagrangian; our numerical values are smaller than the existing estimations.

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1 Introduction

Final-state interactions (or re-scattering effects) play an important role in the hadronic B decays [1–12]. However, it is very difficult to take them into account in a systematic way due to the nonperturbative nature of the multiparticle dynamics. In practical calculations, we can resort to phenomenological models to overcome the difficulty. The one-particle-exchange model is typical (for example, see [12]); in this picture, the soft re-scattering of the intermediate states in two-body channels with one-particle exchange makes the main contributions. The phenomenological Lagrangian contains many input parameters, which describe the strong couplings among the charmed mesons in the hadronic B decays.

In the following, we write down the relevant phenomenological Lagrangian, which describes the strong interactions of the DDV and D^*DV [12],

$$
\begin{aligned}
\mathcal{L} = & \, \mathrm{i}g_{DDV} D_i \big(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu\big) D_j V_{ij}^\mu + 2 f_{D^*DV} \epsilon_{\mu\nu\alpha\beta} \partial^\mu V_{ij}^\nu \\
& \times \big[D_i (\overrightarrow{\partial}^\alpha - \overleftarrow{\partial}^\alpha) D_j^{*\beta} - D_i^{*\beta}(\overrightarrow{\partial}^\alpha - \overleftarrow{\partial}^\alpha) D_j \big] , \\
D = & \, \big(D^0, D^+, D_s \big) , \\
D^* = & \, \big(D^{*0}, D^{*+}, D_s^* \big) , \\
V = & \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} .
\end{aligned} \tag{1}
$$

The strong coupling constants g_{DDV} and f_{D^*DV} in the phenomenological Lagrangian can be related to the basic parameters β and λ in the heavy quark effective Lagrangian (one can consult [13] for the heavy quark effective Lagrangian and relevant parameters.[1]),

$$
\begin{aligned}
g_{DDV} &= \frac{\beta g_V}{\sqrt{2}} , \\
f_{D^*DV} &= \frac{\lambda g_V}{\sqrt{2}} ,
\end{aligned} \tag{2}
$$

where $g_V = 5.8$ from the vector meson dominance theory [14, 15].

In this article, we study the strong coupling constants g_{DDV} and f_{D^*DV} with the light-cone QCD sum rules [16–22]. The strong coupling constants $g_{BB\rho}$, $g_{DD\rho}$, $f_{B^*B\rho}$ and $f_{D^*D\rho}$ have been calculated with the light-cone QCD sum rules in [23][2].

The light-cone QCD sum rules carry out an operator product expansion near the light-cone, $x^2 \approx 0$, instead of at short distance, $x \approx 0$, while the nonperturbative matrix elements are parameterized by the light-cone distribution amplitudes (which are classified according to their twists) instead of the vacuum condensates [16–22]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [24–27].

The article is arranged as follows: in Sect. 2, we derive the strong coupling constants g_{DDV} and f_{D^*DV} with the light-cone QCD sum rules; in Sect. 3, we present the numerical result and a discussion, and in Sect. 4 our conclusions.

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[1] We have $\mathcal{L} = \mathrm{i}\langle H_b v^\mu D_\mu^{ba} \bar{H}_a \rangle + \mathrm{i}\beta\langle H_b v^\mu (\mathcal{V}_\mu - \rho_\mu)_{ba} \bar{H}_a \rangle + \mathrm{i}\lambda\langle H_b \sigma^{\mu\nu} F_{\mu\nu}(\rho)_{ba} \bar{H}_a \rangle .$

[2] I failed to take notice of that work at the beginning.

2 Strong coupling constants g_{DDV} and f_{D^*DV} with light-cone QCD sum rules

We study the strong coupling constants g_{DDV} and f_{D^*DV} with the two-point correlation functions $\Pi_{ij}(p,q)$ and $\Pi_\mu^{ij}(p,q)$,

$$\Pi_{ij}(p,q) = \mathrm{i} \int \mathrm{d}^4 x \, \mathrm{e}^{-\mathrm{i}q\cdot x} \langle 0 | T\{J_i(0)J_j^+(x)\} | V_{ij}(p)\rangle \,, \tag{3}$$

$$\Pi_\mu^{ij}(p,q) = \mathrm{i} \int \mathrm{d}^4 x \, \mathrm{e}^{-\mathrm{i}q\cdot x} \langle 0 | T\{J_\mu^i(0)J_j^+(x)\} | V_{ij}(p)\rangle \,, \tag{4}$$

$$J_i(x) = \bar{q}_i(x)\mathrm{i}\gamma_5 c(x) \,,$$
$$J_\mu^i(x) = \bar{q}_i(x)\gamma_\mu c(x) \,, \tag{5}$$

where the currents $J_i(x)$ interpolate the pseudoscalar mesons D^0, D^+ and D_s, and the currents $J_\mu^i(x)$ interpolate the vector mesons D^{*0}, D^{*+} and D_s^*. The external states ρ, K^* and ϕ have the four momentum p_μ with $p^2 = m_\rho^2$, $m_{K^*}^2$ and m_ϕ^2, respectively.

According to the basic assumption of current–hadron duality in the QCD sum rules [24–27], we can insert a complete series of intermediate states with the same quantum numbers as the current operators $J_i(x)$ and $J_\mu^i(x)$ into the correlation functions $\Pi_{ij}(p,q)$ and $\Pi_\mu^{ij}(p,q)$ to obtain the hadronic representation. After isolating the ground-state contributions from the pole terms of the mesons D_i and D_i^*, we get the following results:

$$\Pi_{ij}(p,q)$$
$$= \frac{f_{D_i} f_{D_j} M_{D_i}^2 M_{D_j}^2 g_{D_i D_j V_{ij}}}{(m_i + m_c)(m_j + m_c)\{M_{D_i}^2 - (q+p)^2\}\{M_{D_j}^2 - q^2\}} 2\epsilon\cdot q$$
$$+ \cdots$$
$$= \Pi_{ij}^P(p,q)\epsilon\cdot q + \cdots \,, \tag{6}$$

$$\Pi_\mu^{ij}(p,q)$$
$$= \frac{f_{D_i^*} f_{D_j} M_{D_i^*} M_{D_j}^2 g_{D_i^* D_j V_{ij}}}{(m_j + m_c)\{M_{D_i^*}^2 - (q+p)^2\}\{M_{D_j}^2 - q^2\}} 4\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha q^\beta$$
$$+ \cdots$$
$$= \Pi_{ij}^V(p,q)\epsilon_{\mu\nu\alpha\beta}\epsilon^\nu p^\alpha q^\beta + \cdots \,, \tag{7}$$

where the following definitions for the weak decay constants have been used:

$$\langle 0 | J_i(0) | D_i(p)\rangle = \frac{f_{D_i} M_{D_i}^2}{m_i + m_c} \,,$$
$$\langle 0 | J_\mu^i(0) | D_i^*(p)\rangle = f_{D_i^*} M_{D_i^*} \epsilon_\mu \,. \tag{8}$$

In (6) and (7), we have not shown the contributions from the high resonances and continuum states explicitly, as they are suppressed due to the double Borel transformation.

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi_{ij}(p,q)$ and $\Pi_\mu^{ij}(p,q)$ in perturbative QCD theory. The calculations

are performed at large spacelike momentum regions, $(q+p)^2 \ll 0$ and $q^2 \ll 0$, which correspond to a small light-cone distance $x^2 \approx 0$, as required by the validity of the operator product expansion. We write down the propagator of a massive quark in the external gluon field in the Fock–Schwinger gauge first [28],

$$\langle 0 | T\{q_i(x_1)\bar{q}_j(x_2)\} | 0\rangle = \mathrm{i} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \mathrm{e}^{-\mathrm{i}k(x_1-x_2)} \frac{\not{k}+m}{k^2 - m^2} + \cdots \,, \tag{9}$$

where we have neglected the contributions from the gluons $G_{\mu\nu}$. The contributions proportional to $G_{\mu\nu}$ can give rise to three-particle (and four-particle) meson distribution amplitudes with a gluon (or quark–antiquark pair) in addition to the two valence quarks; their corrections are usually not expected to play any significant roles.[3] Substituting the above c quark propagator and the corresponding ρ, K^* and ϕ meson light-cone distribution amplitudes into the correlation functions $\Pi_{ij}(p,q)$ and $\Pi_\mu^{ij}(p,q)$ in (3) and (4), and completing the integrals over the variables x and k, we finally obtain the results

$$\Pi_{ij}^P = f_{V_{ij}} m_{V_{ij}} \int_0^1 \mathrm{d}u \frac{\phi_\parallel(u)}{\mathrm{AA}}$$
$$+ \left[f_{V_{ij}}^\perp - f_{V_{ij}} \frac{m_i + m_j}{m_{V_{ij}}} \right] m_c m_{V_{ij}}^2 \int_0^1 \mathrm{d}u \frac{h_\parallel^{(s)}(u)}{\mathrm{AA}^2}$$
$$- \frac{f_{V_{ij}} m_{V_{ij}}^3}{4} \int_0^1 \mathrm{d}u \, A(u) \left[\frac{1}{\mathrm{AA}^2} + \frac{2m_c^2}{\mathrm{AA}^3} \right]$$
$$- 2 f_{V_{ij}} m_{V_{ij}}^3 \int_0^1 \mathrm{d}u \int_0^u \mathrm{d}\tau \int_0^\tau \mathrm{d}t \, C(t) \left[\frac{1}{\mathrm{AA}^2} + \frac{2m_c^2}{\mathrm{AA}^3} \right]$$
$$+ \cdots \,, \tag{10}$$

$$\Pi_{ij}^V = f_{V_{ij}}^\perp \int_0^1 \mathrm{d}u \frac{\phi_\perp(u)}{\mathrm{AA}}$$
$$- \frac{f_{V_{ij}}^\perp m_{V_{ij}}^2}{4} \int_0^1 \mathrm{d}u \, A_\perp(u) \left[\frac{1}{\mathrm{AA}^2} + \frac{2m_c^2}{\mathrm{AA}^3} \right]$$
$$+ \left[f_{V_{ij}} - f_{V_{ij}}^\perp \frac{m_i + m_j}{m_{V_{ij}}} \right] \frac{m_c m_{V_{ij}}}{2} \int_0^1 \mathrm{d}u \frac{g_\perp^{(a)}(u)}{\mathrm{AA}^2}$$
$$+ \cdots \,, \tag{11}$$

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[3] For example, in the decay $B \to \chi_{c0}K$, the factorizable contribution is zero and the non-factorizable contributions from the soft hadronic matrix elements are too small to accommodate the experimental data [35, 36]; the net contributions from the three valence particle light-cone distribution amplitudes to the strong coupling constant $g_{D_{s1}D^*K}$ are rather small, about 20% [37]. The contributions of the three-particle (quark–antiquark–gluon) distribution amplitudes of the mesons are always of minor importance compared to the two-particle (quark–antiquark) distribution amplitudes in the light-cone QCD sum rules. In our previous work, we studied the four form-factors $f_1(Q^2)$, $f_2(Q^2)$, $g_1(Q^2)$ and $g_2(Q^2)$ of $\Sigma \to n$ in the framework of the light-cone QCD sum rules up to twist-6 three-quark light-cone distribution amplitudes and obtained satisfactory results [38]. In the light-cone QCD sum rule approach, we can neglect the contributions from the valence gluons and make relatively rough estimations.

where

$$\text{AA} = m_c^2 - (q+up)^2.$$

In the calculation, the two-particle vector meson light-cone distribution amplitudes have been used [29–34]; the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules [29–34]. In this article, the energy scale μ is chosen to be $\mu = 1\,\text{GeV}$.

Now we perform the double Borel transformation with respect to the variables $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -q^2$ for the correlation functions Π_{ij}^P and Π_{ij}^V in (6) and (7), and obtain the analytical expressions of the invariant functions in the hadronic representation:

$$B_{M_2^2} B_{M_1^2} \Pi_{ij}^P = \frac{2 g_{D_i D_j V_{ij}} f_{D_i} f_{D_j} M_{D_i}^2 M_{D_j}^2}{(m_c + m_i)(m_c + m_j) M_1^2 M_2^2}$$
$$\times \exp\left[-\frac{M_{D_i}^2}{M_1^2} - \frac{M_{D_j}^2}{M_2^2} \right] + \cdots, \quad (12)$$

$$B_{M_2^2} B_{M_1^2} \Pi_{ij}^V = \frac{4 f_{D_i^* D_j V_{ij}} f_{D_i^*} f_{D_j} M_{D_i^*} M_{D_j}^2}{(m_c + m_j) M_1^2 M_2^2}$$
$$\times \exp\left[-\frac{M_{D_i^*}^2}{M_1^2} - \frac{M_{D_j}^2}{M_2^2} \right] + \cdots, \quad (13)$$

where we have not shown the contributions from the high resonances and continuum states explicitly for simplicity.

In order to match the duality regions below the thresholds s_0 and s_0' for the interpolating currents, we can express the correlation functions Π_{ij}^P and Π_{ij}^V at the level of quark–gluon degrees of freedom into the following form:

$$\Pi_{ij}^{P(V)} = \int \mathrm{d}s \int \mathrm{d}s' \frac{\rho_{ij}(s,s')}{\{s - (q+p)^2\}\{s' - q^2\}}, \quad (14)$$

where the $\rho_{ij}(s,s')$ are spectral densities, and then perform the double Borel transformation with respect to the variables Q_1^2 and Q_2^2 directly. However, the analytical expressions of the spectral densities $\rho_{ij}(s,s')$ are hard to obtain; we have to resort to some approximations. As the contributions from the higher twist terms are suppressed by more powers of $\frac{1}{m_c^2 - (q+up)^2}$ (or $\frac{1}{M^2}$), the net contributions of the twist-3 and twist-4 terms are of minor importance (also see the sum rules for the strong coupling constants G_S ($D_{s0}D_s^*\phi$) and G_A ($D_{s1}D_s\phi$) in [39]). The continuum subtractions will not affect the results remarkably. The dominating contributions come from the two-particle twist-2 terms involving $\phi_\parallel(u)$ and $\phi_\perp(u)$. We perform the same trick as [28, 40] and expand the amplitudes $\phi_\parallel(u)$ and $\phi_\perp(u)$ in terms of polynomials of $1-u$,

$$\phi(u) = \sum_{k=0}^{N} b_k (1-u)^k = \sum_{k=0}^{N} b_k \left(\frac{s - m_c^2}{s - q^2} \right)^k, \quad (15)$$

then introduce the variable s', and the spectral density is obtained.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions Π_{ij}^P and Π_{ij}^V at the level of quark–gluon degrees of freedom. The masses of the charmed mesons are $M_D = 1.87\,\text{GeV}$, $M_{D_s} = 1.97\,\text{GeV}$, $M_{D^*} = 2.010\,\text{GeV}$ and $M_{D_s^*} = 2.112\,\text{GeV}$. We have

$$\frac{M_{D^*}}{M_{D^*} + M_{D_s^*}} \approx 0.49, \qquad \frac{M_D}{M_D + M_{D^*}} \approx 0.48,$$
$$\frac{M_{D_s}}{M_{D_s} + M_{D^*}} \approx 0.49, \qquad \frac{M_{D_s}}{M_{D_s} + M_{D_s^*}} \approx 0.48, \quad (16)$$

and there exist overlapping working windows for the two Borel parameters M_1^2 and M_2^2; it is convenient to take the value $M_1^2 = M_2^2$. We introduce the threshold parameters s_0 and make the simple replacement

$$\mathrm{e}^{-\frac{m_c^2 + u_0(1-u_0)m_{\rho,K^*,\phi}^2}{M^2}} \to \mathrm{e}^{-\frac{m_c^2 + u_0(1-u_0)m_{\rho,K^*,\phi}^2}{M^2}} - \mathrm{e}^{-\frac{s_{\rho,K^*,\phi}^0}{M^2}}$$

to subtract the contributions from the high resonances and continuum states [28]. Finally, we obtain the sum rules for the strong coupling constants g_{DDV} and f_{D^*DV},

$$2 g_{D_i D_j V_{ij}} \frac{f_{D_i} f_{D_j} M_{D_i}^2 M_{D_j}^2}{(m_c + m_i)(m_c + m_j)} \exp\left\{ -\frac{M_{D_i}^2}{M_1^2} - \frac{M_{D_j}^2}{M_2^2} \right\}$$
$$= f_{V_{ij}} m_{V_{ij}} M^2 \phi_\parallel(u_0)$$
$$\times \left\{ \exp\left[-\frac{m_c^2 + u_0(1-u_0)m_{V_{ij}}^2}{M^2} \right] - \exp\left[-\frac{s_{V_{ij}}^0}{M^2} \right] \right\}$$
$$+ \exp\left[-\frac{m_c^2 + u_0(1-u_0)m_{V_{ij}}^2}{M^2} \right]$$
$$\times \left\{ \left[f_{V_{ij}}^\perp - f_{V_{ij}} \frac{m_i + m_j}{m_{V_{ij}}} \right] m_c m_{V_{ij}}^2 h_\parallel^{(s)}(u_0) \right.$$
$$- \frac{f_{V_{ij}} m_{V_{ij}}^3 A(u_0)}{4} \left[1 + \frac{m_c^2}{M^2} \right]$$
$$\left. - 2 f_{V_{ij}} m_{V_{ij}}^3 \int_0^{u_0} \mathrm{d}\tau \int_0^\tau \mathrm{d}t\, C(t) \left[1 + \frac{m_c^2}{M^2} \right] \right\}, \quad (17)$$

$$4 f_{D_i^* D_j V_{ij}} \frac{f_{D_i^*} f_{D_j} M_{D_i^*} M_{D_j}^2}{m_c + m_j} \exp\left\{ -\frac{M_{D_i^*}^2}{M_1^2} - \frac{M_{D_j}^2}{M_2^2} \right\}$$
$$= f_{V_{ij}}^\perp M^2 \phi_\perp(u_0)$$
$$\times \left\{ \exp\left[-\frac{m_c^2 + u_0(1-u_0)m_{V_{ij}}^2}{M^2} \right] - \exp\left[-\frac{s_{V_{ij}}^0}{M^2} \right] \right\}$$
$$+ \exp\left[-\frac{m_c^2 + u_0(1-u_0)m_{V_{ij}}^2}{M^2} \right]$$
$$\times \left\{ \left[f_{V_{ij}} - f_{V_{ij}}^\perp \frac{m_i + m_j}{m_{V_{ij}}} \right] \frac{m_c m_{V_{ij}} g_\perp^{(a)}(u_0)}{2} \right.$$
$$\left. - \frac{f_{V_{ij}}^\perp m_{V_{ij}}^2 A_\perp(u_0)}{4} \left[1 + \frac{m_c^2}{M^2} \right] \right\}, \quad (18)$$

where

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \quad (19)$$

3 Numerical result and discussion

The input parameters are taken as [29–34]

$$m_s = (0.14 \pm 0.01)\,\text{GeV}, \qquad m_c = (1.35 \pm 0.10)\,\text{GeV},$$
$$m_u = m_d = (0.0056 \pm 0.0016)\,\text{GeV},$$
$$f_\rho = (0.216 \pm 0.003)\,\text{GeV}, \qquad f_\rho^\perp = (0.165 \pm 0.009)\,\text{GeV},$$
$$f_{K^*} = (0.220 \pm 0.005)\,\text{GeV}, \qquad f_{K^*}^\perp = (0.185 \pm 0.010)\,\text{GeV},$$
$$f_\phi = (0.215 \pm 0.005)\,\text{GeV}, \qquad f_\phi^\perp = (0.186 \pm 0.009)\,\text{GeV},$$
$$m_\rho = 0.775\,\text{GeV}, \qquad m_{K^*} = 0.892\,\text{GeV},$$
$$m_\phi = 1.02\,\text{GeV}, \qquad \zeta_4 = 0.15 \pm 0.10,$$
$$\zeta_4^T = 0.10 \pm 0.05, \qquad \widetilde{\zeta}_4^T = -0.10 \pm 0.05.$$

The parameters in the two-particle twist-2 and twist-3 light-cone distribution amplitudes are shown in Table 1 [29–34].

The values of the decay constants f_D and f_{D_s} vary in a large range from different approaches, for example, the potential model, QCD sum rules and lattice QCD, etc. [41–43]. For the decay constant f_D, we take the experimental data from the CLEO Collaboration, $f_D = (0.223 \pm 0.017)\,\text{GeV}$ [44–46]. If we take the value $f_{D_s} = (0.274 \pm 0.013)\,\text{GeV}$ from the CLEO Collaboration, the SU(3) breaking effect is rather large, $\frac{f_{D_s}}{f_D} = 1.23$, while most theoretical estimations indicate $\frac{f_{D_s}}{f_D} \approx 1.1$. In this article, we take the value $\frac{f_{D_s}}{f_D} = 1.1$. For the decay constants f_{D^*} and $f_{D_s^*}$, we take the central values from a lattice simulation [47], $f_{D^*} = (0.23 \pm 0.02)\,\text{GeV}$ and $f_{D_s^*} = (0.25 \pm 0.02)\,\text{GeV}$,

$$\frac{f_{D_s^*}}{f_{D^*}} \approx \frac{f_{D_s}}{f_D} = 1.1. \tag{20}$$

Table 1. The parameters in the twist-2 and twist-3 light-cone distribution amplitudes (taken from [34])

	ρ	K^*	ϕ
$a_1^{\parallel}$	0	0.03(2)	0
$a_1^{\perp}$	0	0.04(3)	0
$a_2^{\parallel}$	0.15(7)	0.11(9)	0.18(8)
$a_2^{\perp}$	0.14(6)	0.10(8)	0.14(7)
$\zeta_{3V}^{\parallel}$	0.030(10)	0.023(8)	0.024(8)
$\widetilde{\lambda}_{3V}^{\parallel}$	0	0.035(15)	0
$\widetilde{\omega}_{3V}^{\parallel}$	−0.09(3)	−0.07(3)	−0.045(15)
$\kappa_{3V}^{\parallel}$	0	0.000(1)	0
$\omega_{3V}^{\parallel}$	0.15(5)	0.10(4)	0.09(3)
$\lambda_{3V}^{\parallel}$	0	−0.008(4)	0
$\kappa_{3V}^{\perp}$	0	0.003(3)	0
$\omega_{3V}^{\perp}$	0.55(25)	0.3(1)	0.20(8)
$\lambda_{3V}^{\perp}$	0	−0.025(20)	0

Table 2. Threshold parameters for the strong coupling constants g_{DDV} and f_{D^*DV}

	g_{DDV}	f_{D^*DV}
s_ρ^0 (GeV^2)	6.0 ± 0.5	6.5 ± 0.5
$s_{K^*}^0$ (GeV^2)	6.3 ± 0.5	7.0 ± 0.5
s_ϕ^0 (GeV^2)	6.3 ± 0.5	7.0 ± 0.5

The duality threshold parameters s_0 are shown in Table 2; the numerical (central) values of s_0 are taken from the QCD sum rules for the masses of the pseudoscalar mesons D^0, D^+ and D_s and the vector mesons D^{*0}, D^{*+} and D_s^* [48]. In this article, we take the uncertainties for the threshold parameters s_0 to be $0.5\,\text{GeV}^2$ for simplicity. The Borel parameters are chosen as $M_1^2 = M_2^2$ and $M^2 = (3–7)\,\text{GeV}^2$. In these regions, the values of the strong coupling constants g_{DDV} and f_{D^*DV} are rather stable.

In the limit of a large Borel parameter M^2, the strong coupling constants g_{DDV} and f_{D^*DV} take up the following behavior,

$$g_{D_iD_jV_{ij}} \propto \frac{M^2 f_{V_{ij}}\phi_{\parallel}(u_0)}{f_{D_i}f_{D_j}} \propto \frac{M^2 f_{V_{ij}}a_2^{\parallel}}{f_{D_i}f_{D_j}},$$
$$f_{D_i^*D_jV_{ij}} \propto \frac{M^2 f_{V_{ij}}^\perp\phi_{\perp}(u_0)}{f_{D_i^*}f_{D_j}} \propto \frac{M^2 f_{V_{ij}}^\perp a_2^{\perp}}{f_{D_i^*}f_{D_j}}. \tag{21}$$

It is not unexpected that the contributions from the twist-2 light-cone distribution amplitudes $\phi_{\parallel}(u)$ and $\phi_{\perp}(u)$ are greatly enhanced by the large Borel parameter M^2; (large) uncertainties of the relevant parameters presented in the above equations have a significant impact on the numerical results.

Taking into account all the uncertainties, we finally obtain the numerical values for the strong coupling constants g_{DDV} and f_{D^*DV}, which are shown in Figs. 1 and 2,

$$g_{DD\rho} = 1.31 \pm 0.29,$$
$$g_{DD_sK^*} = 1.61 \pm 0.32,$$
$$g_{D_sD_s\phi} = 1.45 \pm 0.34,$$
$$f_{D^*D\rho} = (0.89 \pm 0.15)\,\text{GeV}^{-1},$$
$$f_{D^*D_sK^*} = (1.01 \pm 0.20)\,\text{GeV}^{-1},$$
$$f_{D_s^*D_s\phi} = (0.82 \pm 0.16)\,\text{GeV}^{-1}. \tag{22}$$

Taking the replacements $g_{DD\rho} \to \frac{g_{DD\rho}}{2}$ and $f_{D^*D\rho} \to \frac{f_{D^*D\rho}}{4}$ in (1), we can obtain the same definitions for the strong coupling constants in [23]. Our numerical values $g_{DD\rho} = 2.62 \pm 0.58$ and $f_{D^*D\rho} = (3.56 \pm 0.60)\,\text{GeV}^{-1}$ are compatible with the predictions $g_{DD\rho} = 3.81 \pm 0.88$ and $f_{D^*D\rho} = (4.17 \pm 1.04)\,\text{GeV}^{-1}$ in [23]. In [23], the authors take much smaller values for the decay constants of the charmed mesons than the present work. It is not unexpected that the numerical values are different from each other; see (21).
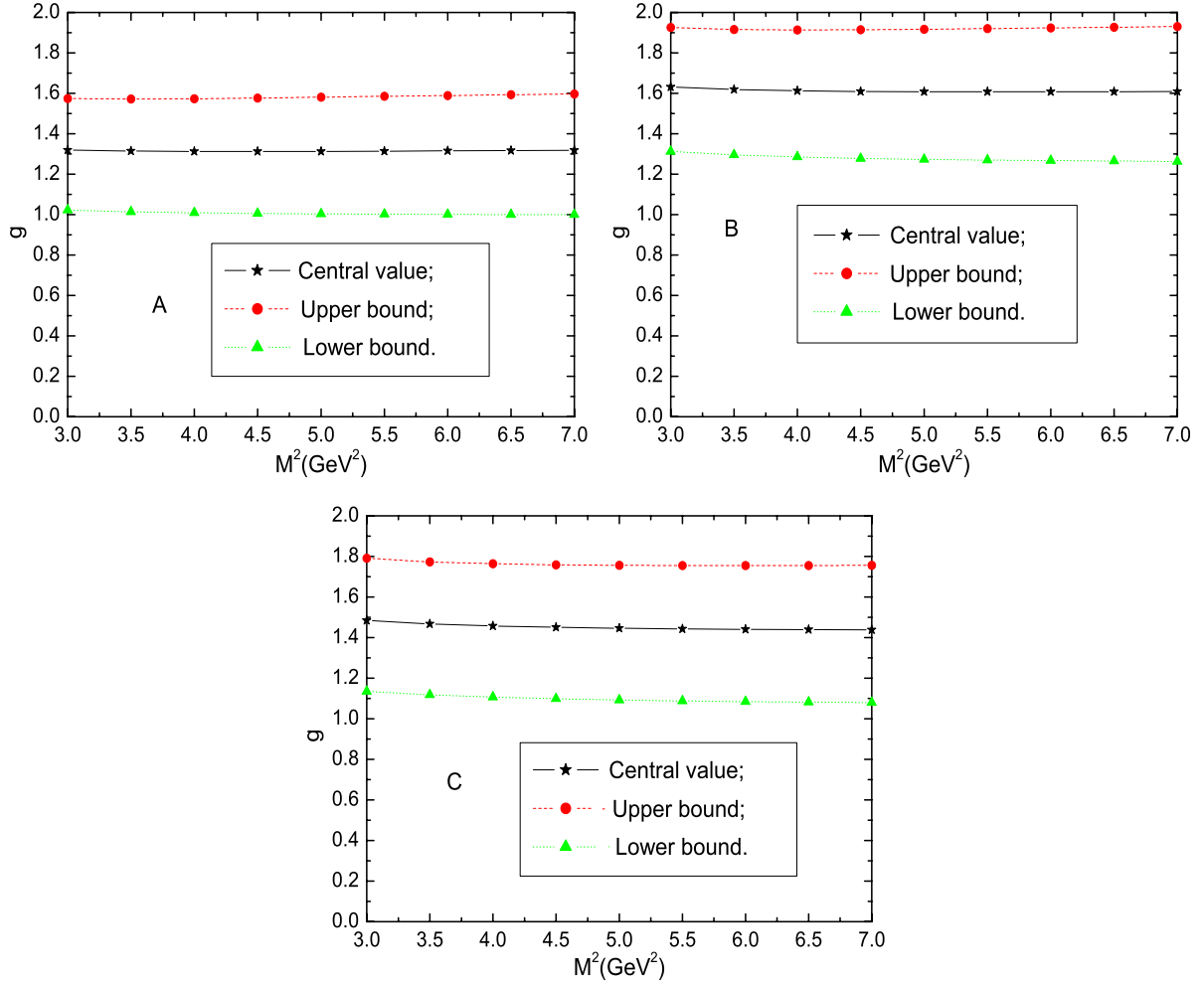
Fig. 1. $g_{DD\rho}$ (**A**), $g_{DD_sK^*}$ (**B**) and $g_{D_sD_s\phi}$ (**C**) with the Borel parameter M^2

The average values of the strong coupling constants are about

$$g_{DDV} = 1.46 \pm 0.32,$$
$$f_{D^*DV} = (0.91 \pm 0.17)\,\text{GeV}^{-1}. \tag{23}$$

The corresponding basic parameters β and λ in the heavy quark effective theory are listed in Tables 3 and 4, respectively. The parameter β can be estimated with the vector meson dominance theory, as we show in the appendix; it is presented in Table 3. The basic parameter λ relates to the form-factor $V(q^2)$ of the hadronic transitions $\langle V|\bar{q}\gamma_\mu(1-\gamma_5)b|B\rangle$ and $\langle V|\bar{q}\sigma_{\mu\nu}(1+\gamma_5)b|B\rangle$, which can be calculated with the light-cone sum rules and lattice QCD. With the assumption that the form-factor $V(q^2)$ at $q^2 = q^2_{\max} = (M_B - M_V)^2$ is dominated by the nearest low-lying vector meson pole, we can obtain the values of λ [49–52], which are presented in Table 4. From Tables 3 and 4, we can see that our numerical values are much smaller.

One possibility for the large discrepancies may be that the vector meson dominance theory overestimates the values of βg_V and λg_V; the other possibility may be

Table 3. Numerical values of the parameter β

β	Reference
0.9	[49]
0.36 ± 0.08	this work

Table 4. Numerical values of the parameter λ

$|\lambda|$ (GeV^{-1})	Reference
0.56	[49]
0.63 ± 0.17	[50–52]
0.22 ± 0.04	this work

the shortcomings of the light-cone QCD sum rules. We can borrow some idea from the strong coupling constant $g_{D^*D\pi}$; the central value ($g_{D^*D\pi} = 12.5$ or $g_{D^*D\pi} = 10.5$ with the radiative corrections included) from the light-cone QCD sum rules is too small to take into account the value
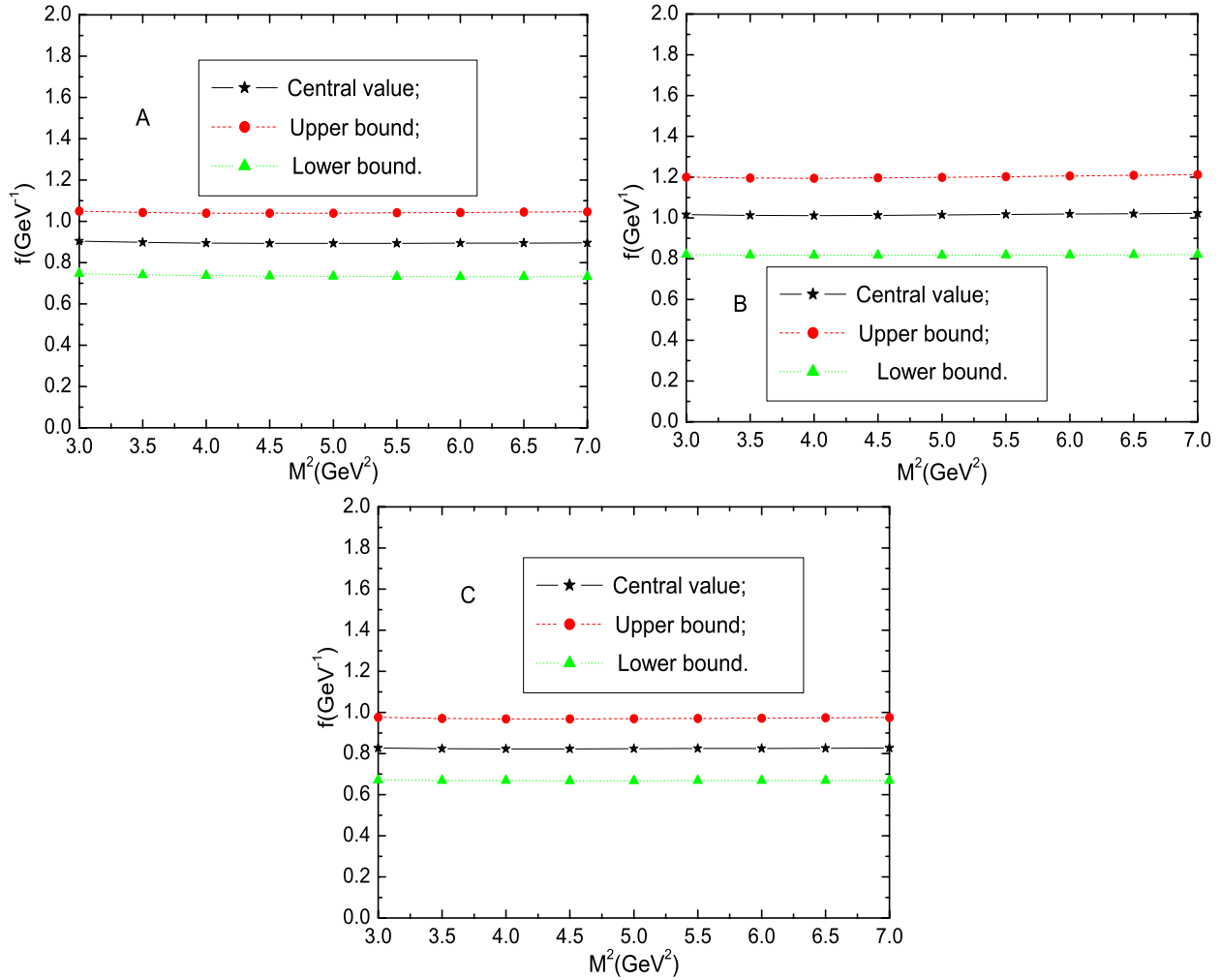
Fig. 2. $f_{D^*D\rho}$ (**A**), $f_{D^*D_sK^*}$ (**B**) and $f_{D_s^*D_s\phi}$ (**C**) with the Borel parameter M^2

($g_{D^*D\pi} = 17.9$) from the experimental data [28, 53, 54]. It has been noted that the simple quark–hadron duality ansatz that works in the one-variable dispersion relation might be too crude for the double dispersion relation [55]. As in [54], we can postpone the threshold parameters s_0 to larger values to include the contributions from a radial excitation (D' or $D^{*\prime}$) to the hadronic spectral densities, with the additional assumption for the values of $g_{D'DV}$, $f_{D^{*\prime}DV}$ and $f_{D^*D'V}$, we can then improve the values of g_{DDV} and f_{D^*DV} and smear the discrepancies between our values and the predictions with the vector meson dominance theory. It is somewhat a matter of fine-tuning.

Naively, we can expect smaller values of the strong coupling constants to lead to smaller final-state re-scattering effects in the hadronic B decays. For example, the contributions from the re-scattering mechanism in the decay

$$B \to D^*\rho \to D\pi$$

can occur through exchange of D^* (or D) in the t channel for the sub-process $D^*\rho \to D\pi$ [12]. The amplitude of the re-scattering Feynman diagrams is proportional to

$$C_1 g_{D^*D^*\pi} f_{D^*D\rho} + C_2 g_{D^*D\pi} g_{DD\rho}, \qquad (24)$$

where the C_i are some coefficients.

4 Conclusion

In this article, we study the vertices DDV and D^*DV with the light-cone QCD sum rules. The strong coupling constants g_{DDV} and f_{D^*DV} play an important role in understanding the final-state re-scattering effects in the hadronic B decays. They are related to the basic parameters β and λ in the heavy quark effective Lagrangian; the numerical values are much smaller than the existing estimations based on the assumption of vector meson dominance. If the predictions from the light-cone QCD sum rules are robust, the final-state re-scattering effects may be overestimated in the hadronic B decays.

Appendix

4.1 Two-particle vector meson light-cone distribution amplitudes

The light-cone distribution amplitudes of the K^* meson are defined by

$$\langle 0|\bar{u}(0)\gamma_\mu s(x)|K^*(p)\rangle$$
$$= p_\mu f_{K^*} m_{K^*} \frac{\epsilon \cdot x}{p \cdot x} \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} \left\{ \phi_\|(u) + \frac{m_{K^*}^2 x^2}{16} A(u) \right\}$$
$$+ \left[\epsilon_\mu - p_\mu \frac{\epsilon \cdot x}{p \cdot x} \right] f_{K^*} m_{K^*} \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} g_\perp^{(v)}(u)$$
$$- \frac{1}{2} x_\mu \frac{\epsilon \cdot x}{(p \cdot x)^2} f_{K^*} m_{K^*}^3 \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} C(u) ,$$

$$\langle 0|\bar{u}(0)s(x)|K^*(p)\rangle$$
$$= \frac{\mathrm{i}}{2} \left[f_{K^*}^\perp - f_{K^*} \frac{m_u + m_s}{m_{K^*}} \right] m_{K^*}^2 \epsilon \cdot x \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} h_\|^{(s)}(u) ,$$

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu} s(x)|K^*(p)\rangle$$
$$= \mathrm{i}[\epsilon_\mu p_\nu - \epsilon_\nu p_\mu] f_{K^*}^\perp$$
$$\times \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} \left\{ \phi_\perp(u) + \frac{m_{K^*}^2 x^2}{16} A_\perp(u) \right\}$$
$$+ \mathrm{i}[p_\mu x_\nu - p_\nu x_\mu] f_{K^*}^\perp m_{K^*}^2 \frac{\epsilon \cdot x}{(p \cdot x)^2} \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} B_\perp(u)$$
$$+ \mathrm{i}\frac{1}{2}[\epsilon_\mu x_\nu - \epsilon_\nu x_\mu] f_{K^*}^\perp m_{K^*}^2 \frac{1}{p \cdot x} \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} C_\perp(u) ,$$

$$\langle 0|\bar{u}(0)\gamma_\mu \gamma_5 s(x)|K^*(p)\rangle$$
$$= -\frac{1}{4} \left[f_{K^*} - f_{K^*}^\perp \frac{m_u + m_s}{m_{K^*}} \right] m_{K^*} \epsilon_{\mu\nu\alpha\beta} \epsilon^\nu p^\alpha x^\beta$$
$$\times \int_0^1 \mathrm{d}u\, \mathrm{e}^{-\mathrm{i}up \cdot x} g_\perp^{(a)}(u) . \tag{25}$$

The light-cone distribution amplitudes of the K^* meson are parameterized as

$$\phi_\|(u,\mu) = 6u(1-u)\left\{ 1 + a_1^\| 3\xi + a_2^\| \frac{3}{2}(5\xi^2 - 1) \right\} ,$$

$$\phi_\perp(u,\mu) = 6u(1-u)\left\{ 1 + a_1^\perp 3\xi + a_2^\perp \frac{3}{2}(5\xi^2 - 1) \right\} ,$$

$$g_\perp^{(v)}(u,\mu) = \frac{3}{4}(1+\xi^2) + a_1^\| \frac{3}{2}\xi^3 + \left\{ \frac{3}{7}a_2^\| + 5\zeta_3^\| \right\}(3\xi^2 - 1)$$
$$+ \left\{ 5\kappa_3^\| - \frac{15}{16}\lambda_3^\| + \frac{15}{8}\widetilde{\lambda}_3^\| \right\}\xi(5\xi^2 - 3)$$
$$+ \left\{ \frac{9}{112}a_2^\| + \frac{15}{32}\omega_3^\| - \frac{15}{64}\widetilde{\omega}_3^\| \right\}(3 - 30\xi^2 + 35\xi^4) ,$$

$$g_\perp^{(a)}(u,\mu) = 6u\bar{u}\left\{ 1 + \left(\frac{1}{3}a_1^\| + \frac{20}{9}\kappa_3^\| \right)C_1^{\frac{3}{2}}(\xi) \right.$$
$$+ \left(\frac{1}{6}a_2^\| + \frac{10}{9}\zeta_3^\| + \frac{5}{12}\omega_3^\| - \frac{5}{24}\widetilde{\omega}_3^\| \right)C_2^{\frac{3}{2}}(\xi)$$
$$\left. + \left(\frac{1}{4}\widetilde{\lambda}_3^\| - \frac{1}{8}\lambda_3^\| \right)C_3^{\frac{3}{2}}(\xi) \right\}$$

$$h_\|^{(s)}(u,\mu) = 6u\bar{u}\left\{ 1 + \left(\frac{a_1^\perp}{3} + \frac{5}{3}\kappa_3^\perp \right)C_1^{\frac{3}{2}}(\xi) \right.$$
$$\left. + \left(\frac{a_2^\perp}{6} + \frac{5}{18}\omega_3^\perp \right)C_2^{\frac{3}{2}}(\xi) - \frac{1}{20}\lambda_3^\perp C_3^{\frac{3}{2}}(\xi) \right\} ,$$

$$h_\|^{(t)}(u,\mu) = 3\xi^2 + \frac{3}{2}a_1^\perp \xi(3\xi^2 - 1) + \frac{3}{2}a_2^\perp \xi^2(5\xi^2 - 3)$$
$$+ \frac{5}{8}\omega_3^\perp(3 - 30\xi^2 + 35\xi^4)$$
$$+ \left(\frac{15}{2}\kappa_3^\perp - \frac{3}{4}\lambda_3^\perp \right)\xi(5\xi^2 - 3) ,$$

$$g_3(u,\mu) = 1 + \left\{ -1 - \frac{2}{7}a_2^\| + \frac{40}{3}\zeta_3^\| - \frac{20}{3}\zeta_4 \right\}C_2^{\frac{1}{2}}(\xi)$$
$$+ \left\{ -\frac{27}{28}a_2^\| + \frac{5}{4}\zeta_3^\| - \frac{15}{16}\widetilde{\omega}_3^\| - \frac{15}{8}\omega_3^\| \right\}C_4^{\frac{1}{2}}(\xi) ,$$

$$h_3(u,\mu) = 1 + \left\{ -1 + \frac{3}{7}a_2^\perp - 10(\zeta_4^T + \widetilde{\zeta}_4^T) \right\}C_2^{\frac{1}{2}}(\xi)$$
$$+ \left\{ -\frac{3}{7}a_2^\perp - \frac{5}{4}\omega_3^\perp \right\}C_4^{\frac{1}{2}}(\xi) ,$$

$$A(u,\mu) = 30u^2\bar{u}^2\left\{ \frac{4}{5} + \frac{4}{105}a_2^\| + \frac{8}{9}\zeta_3^\| + \frac{20}{9}\zeta_4 \right\} ,$$

$$A_\perp(u,\mu) = 30u^2\bar{u}^2\left\{ \frac{2}{5} + \frac{4}{35}a_2^\perp + \frac{4}{3}\zeta_4^T - \frac{8}{3}\widetilde{\zeta}_4^T \right\} ,$$

$$C(u,\mu) = g_3(u,\mu) + \phi_\|(u,\mu) - 2g_\perp^{(v)}(u,\mu) ,$$

$$B_\perp(u,\mu) = h_\|^{(t)}(u,\mu) - \frac{1}{2}\phi_\perp(u,\mu) - \frac{1}{2}h_3(u,\mu) ,$$

$$C_\perp(u,\mu) = h_3(u,\mu) - \phi_\perp(u,\mu) , \tag{26}$$

where $\xi = 2u - 1$, and $C_2^{\frac{1}{2}}(\xi)$, $C_4^{\frac{1}{2}}(\xi)$, $C_1^{\frac{3}{2}}(\xi)$, $C_2^{\frac{3}{2}}(\xi)$ and $C_3^{\frac{3}{2}}(\xi)$ are Gegenbauer polynomials. The corresponding light-cone distribution amplitudes for the ρ and ϕ mesons can be obtained with a simple replacement of the nonperturbative parameters.

4.2 Estimation of the basic parameter β

In this appendix, we illustrate the estimation of the basic parameter β with the vector meson dominance theory.

We have

$$f(p^2)(p_1 + p_2)_\mu = \langle D_s(p_1)|\bar{s}(0)\gamma_\mu s(0)|D_s(p_2)\rangle$$
$$= \langle D_s(p_1)\phi(p)|D_s(p_2)\rangle$$
$$\times \frac{\mathrm{i}}{m_\phi^2 - p^2}\langle 0|\bar{s}(0)\gamma_\mu s(0)|\phi(p)\rangle$$
$$= \frac{1}{p^2 - m_\phi^2} f_\phi m_\phi g_{D_s D_s \phi} \epsilon^* \cdot (p_1 + p_2)\epsilon_\mu$$

$$= \frac{1}{p^2 - m_\phi^2} f_\phi m_\phi g_{D_s D_s \phi}(p_1 + p_2)_\nu$$

$$\times \left\{ -g_{\mu\nu} + \frac{(p_1 - p_2)_\mu (p_1 - p_2)_\nu}{(p_1 - p_2)^2} \right\}$$

$$= -\frac{1}{p^2 - m_\phi^2} f_\phi m_\phi g_{D_s D_s \phi}(p_1 + p_2)_\mu$$

$$\tag{27}$$

$$\rightarrow \qquad f(0) = \frac{f_\phi}{m_\phi} g_{D_s D_s \phi} .$$

Take the normalization condition $f(0) = 1$,

$$\rightarrow \quad g_{D_s D_s \phi} = \frac{m_\phi}{f_\phi} ,$$

$$\rightarrow \quad \frac{\beta g_V}{\sqrt{2}} = \frac{m_\phi}{f_\phi} , \quad [49] . \tag{28}$$

If we take into account the contribution from the $2\,^3S_1$ state $\phi(1680)$, the expression would be

$$1 = \frac{f_\phi}{m_\phi} g_{D_s D_s \phi} + \frac{f_{\phi(1680)}}{m_{\phi(1680)}} g_{D_s D_s \phi(1680)} . \tag{29}$$

If the value of $g_{D_s D_s \phi(1680)}$ is positive, a much smaller value of β can be obtained. For example, with the assumption $g_{D_s D_s \phi(1680)} = g_{D_s D_s \phi}$ and $f_{\phi(1680)} = f_\phi$, we can obtain $\frac{\beta g_V}{\sqrt{2}} = \frac{m_\phi m_{\phi(1680)}}{(m_\phi + m_{\phi(1680)}) f_\phi} = \frac{0.62 m_\phi}{f_\phi}$, the value of β listed in Table 3 would be $\beta \approx 0.62 \times 0.9 \approx 0.56$, and our prediction is still much smaller.

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